STRUCTURAL DYNAMIC MODIFICATION USING DISCRETE MODELS

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AbstractStructural dynamic modification techniques are implemented at design stage to achieve dynamic design of mechanical structure. These structures are often well defined in terms of mass, stiffness and damping distribution in the system or they may be obtained in terms of spatial models from the frequency response function of the system using experimental modal analysis. The FRF obtained may be from direct measurement or from updated models. In present study, the SDM on a discrete model are performed to understand the effects of different modifications on a mechanical system and results shows that few modifications lead to a significant change in dynamics of the system while others have very little effect and thus should be avoided.

Index Terms— Structural dynamic modifications, Dynamic design, Frequency response function.

1 INTRODUCTION

STRUTURAL dynamic modifications (SDM) are the techniques using which the dynamic behavior of the structure can be altered (improved) by modifying the parameters of structure such as lumped mass, damper, beams & rigid links etc. this may also be achieved by changing the configuration parameters of the structure (i.e.- orientation of certain links on their connectivity). Such methods which are specially dealt with help of finite element (FE) models are often termed as REANALYSIS of the structure given by (Baldwin and Hutton 1985, Brandon 1990) most of the techniques utilizes dynamic tests at different levels of the SDM and can be easily implemented at user's end.

Due to cut throat competition in the market, there is a high pressure of introducing new products which are becoming more & more complex as for as their mechanical & structural design is concerned. The component as automobiles & aerospace vehicle require very accurate dynamic design (their desired dynamic characteristics must be specific). As the FE analysis is back bone of almost every industry in today's world and the dynamic design is carried out on FE models, the dynamic design has considerable advantage of performance improvement at prototype level thereby reducing the cycle time of design- modification & re-prototyping and initial level of model development and can easily be implemented on personal computer of design engineer who can predict and analyze the change of design parameters such as linkages, rigid masses, dampers, which is the reverse of the above method, and required configuration changes on the structures dynamic characteristics with respect to the initial prototype under given operating conditions.

The second important objective, is the prediction of magnitude and / or location of parameter modification for changing the area of resonant frequencies or improvement of other parameters of dynamic performance Eigen value and Eigen vector sensitivity analysis with respect to the changed parameters can help in achieving this goal. These methods then can be used for the optimal modifications with several available optimization techniques these optimization techniques to achieve SDM are implemented by (Rao 1989) in vibrating environment.

SDM techniques can be classified in two broad groups

(1) Methods which employ a model of structure.

(2) Methods which uses dynamic test data of the structure.

The former techniques use models' physical and / or spatial matrices (such as mass, stiffness & damping parameters or modal model) such as (mode shapes, natural frequency & damping ratios) these parameters can be extracted from test data by experimental modal analysis and are reported by several researchers as (Berman 1975), (Natke 1982 and Ewins 1984). The modal used here can be initial model or an updated one (FE updated model) using FE model updating techniques (Imregun and Visser 1991, Lin, Zhu 2007, Marwala 2010) considering experimental data.

In later techniques of SDM, the experimental data which is obtained from experimental modal analysis is directly used. It is evident that the experimental data is assumed to be error free and the frequency response thus obtained are utilized for SDM purpose.

In this paper, the former technique of SDM is utilized and the results are analyzed & compared using different SDM approaches.

2 MATHEMATICAL MODELLING

2.1 SDM

The dynamic behavior of a linear discrete system of 'n' degree of freedom (dof) can be given by

$$[M](\ddot{x}) + [C](\dot{x}) + [K](x) + i[H](x) = (f) \quad (1)$$

Where, M & K are the mass stiffness and C and H are damping matrices of order nxn and f is force acting and the system of order nx1.

Solving the above equation for Eigen values gives-

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$$[K - \lambda M](x) = 0 \quad (2)$$

If SDM techniques is applied to the system and δ indicate the change in the mass, stiffness and damping matrices of system. Equation (1) can be written in modified form as: -

$$([K] + \delta[K] - \lambda'([M] + \delta[M]))(x) = 0 \quad (3)$$

and the Eigen value solution can be given as-

$$[M]'^{(q)} + [C]'^{(q)} + [K]'^{(q)} + i[H]'^{(q)} = [\phi](f) \quad (4)$$

Here, $[\phi]$ modal matrix

Writing the above equation in modal co-ordinates yields

$$[M]' = [\phi]t[M][\phi] + [\phi]t[\delta M][\phi] \quad (5a)$$
$$[K]' = [\phi]t[K][\phi] + [\phi]t[\delta K][\phi] \quad (5b)$$
$$[C]' = [\phi]t[C][\phi] + [\phi]t[\delta C][\phi] \quad (5c)$$
$$[H]' = [\phi]t[H][\phi] + [\phi]t[\delta H][\phi] \quad (5d)$$

The above solution is given by (Ravi, Nakra and Kundra 1995) the terms in equation 5(a-d) can be obtained by experimental data using experimental modal analysis techniques given by (Ewins 1984) the M, K & C matrices are obviously of reduced size as the experimental dof are always less than the structures dof or FE model dof equation 4 can be used for predicting dynamic behavior of modified structure after SDM is applied the most imported thing to note here is that the coordinates of modifying matrices δ(M, K & C) must be oriented same as that of original M, K & C matrices in order to be added (Kundra 2000) , which needs to be considered during SDM process.

2.2 Mass modification

The factor which determines the mass modification matrix [δM] is the orientation and position on which mass has to be added, which can easily be taken into account by using same spatial configuration as that of [M] & the modified mass matrix can be obtained by inserting m (modifying mass) at diagonal elements of a null matrix and at the location where the motion co-ordinates gets influenced by m.

$$[\delta M] = \begin{bmatrix} +m & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (6)$$

Equation 6 shows the addition of modifying mass at the first coordinate (node) of the original mass matrix of the structure similarly if modifying mass m is shifted from place of mass m2 (due to structural change or shifting of some attached equipment or machine component) to mass m1 the modifying matrix becomes

$$[\delta M] = \begin{bmatrix} +m & 0 & 0 \\ 0 & -m & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (7)$$

Here, -m indicates mass removal from second co-ordinate and its addition to first co-ordinate as +m.

2.3 Damping Modification

The procedure of obtaining the modifying damping matrix H as C (as the case may be) is not a direct procedure as that of mass modification because the damping effects the entire matrix. In case of structural damping modification (due to change in material) the modifying matrix may be written.

$$[\delta H] = a[H] \quad (8)$$

Where, a is desired structural damping. Co-efferent and is average value of identified damping of several modes.

$$a = a' - a_{avg} \quad (9)$$

2.4 Stiffness and Beam modifications

The stiffness modification for simple structure can be an easy task which can be carried out on mass lines modification. The difficulty arises while using beam modification on real structures. The beam modifications or real structures proved that these modifications influence all parameter matrices in complex manner. This is due to the fact that the rotational dof are not easy to measure as well as the stiffness modifications do not alter the matrix uniformly so it is almost impossible to carry out modifications for stiffness matrix in exact and simple way.

The above problem is solved by the making the use of stiffness modification coefficients k given as

$$[k] = \frac{[K]_{mod}}{[K]_{ana}} \quad (10)$$

$$[K]' = k * [K] \quad (11)$$

K- reduced stiffness matrix of analytical model.
K'- reduced stiffness matrix of modified model.

Here $\frac{[K]_{mod}}{[K]_{ana}}$ denotes the simple division of corresponding elements. The modified stiffness matrix of the system may be written in terms of modifying ratios. Here, $k * [K]$ denotes multiplication of corresponding elements

3 NUMERICAL EXAMPLE

Consider a 2-dof system as shown in figure-1. The above SDM techniques are applied one by one and their effects are analyzed.

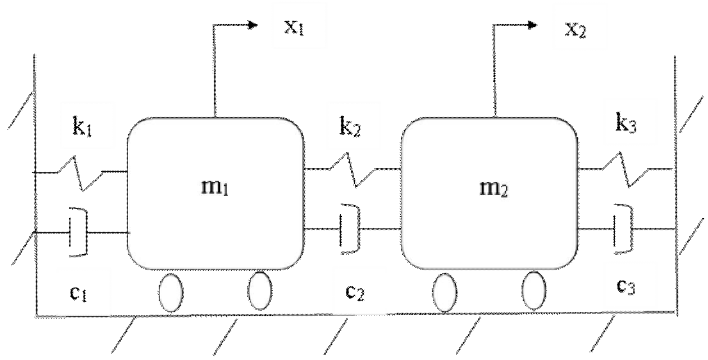


Figure 1: Two Degree of freedom discrete system

The initial values considered in this study are summarized in the following table.

International Journal of Scientific & Engineering Research Volume 8, Issue 7, July-2017
ISSN 2229-5518

Table 1: values of parameters used for two Degree of freedom discrete system

Lumped Mass			Stiffness			Dampers		
m_1 =	1	Kg	K_1=	1	N/m	C_1=	0.001	Ns/m
			K_2=	1		C_2=	0.001	
m_2 =	2		K_3=	1		C_3=	0.001	

Mass Modification- (Mass addition) Consider a small mass 'm' (=0.25 Kg) added to first discrete mass m1, the modifying mass matrix [M] becomes

$$[\delta M] = \begin{bmatrix} +0.25 & 0 \\ 0 & 0 \end{bmatrix} \quad (12)$$

While the other elements are zero. as there is no change in stiffness and damping parameters. The modifying stiffness & damping matrices are zero.

shifting of mass: - suppose a small mass 'm' is shifted from lumped mass m1 to m2, other parameters being same yields the modifying matrix

$$[\delta M] = \begin{bmatrix} +m & 0 \\ 0 & -m \end{bmatrix} \quad (13)$$

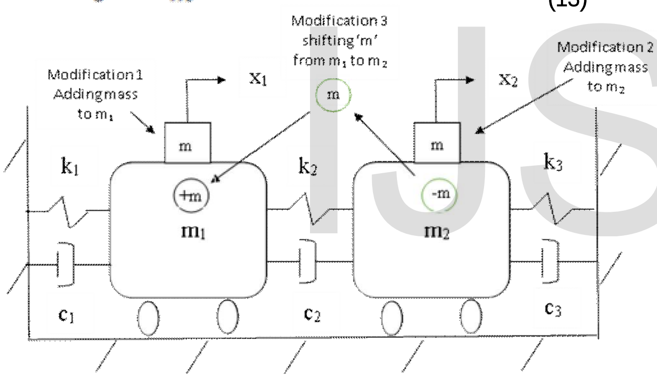


Figure 2: Mass Modification (a) Adding mass on m1 (b) adding mass on m2 (c) Shifting mass from m1 to m2

Stiffness Modification: - Let a stiffing member is added to lumped mass m1 as shown in figure-3. as this is a simple modification on the structure it does not require evaluation of stiffness coefficient describe in section 2.3. Assuming stiffer mass to be negligible effect the modified matrix can be obtained in a similar fashion as that of mass modification.

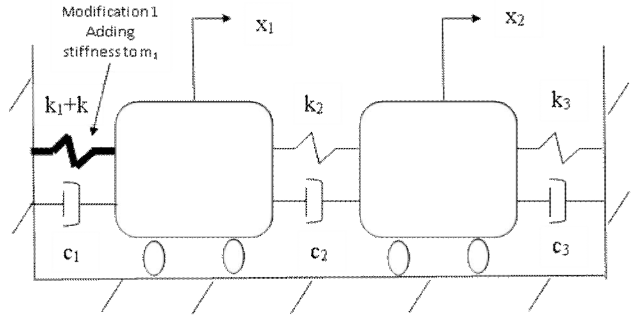


Figure 3: Stiffness Modification (a) Adding stiffener to m1 (b) Adding stiffener to m2 (c) Adding stiffener between m1, m2

Damping Modification: - for the above modified system, the modifying damping matrix may be calculated as that of mass and stiffness matrix by inserting the damping values at corresponding nodes since it is a discrete system.

The above modifications can be readily obtained by reanalysis of the system, however for a simple system with lesser degree of freedoms there is no bar but for complex system and specific modifications the reanalysis may prove time consuming task for this the reanalysis can be done using the relation given below.

$$\delta[\lambda]' = (\sigma + \delta\sigma)^2 = \frac{[\phi]^t[K+\delta K][\phi]}{[\phi]^t[M+\delta M][\phi]} \quad (14)$$

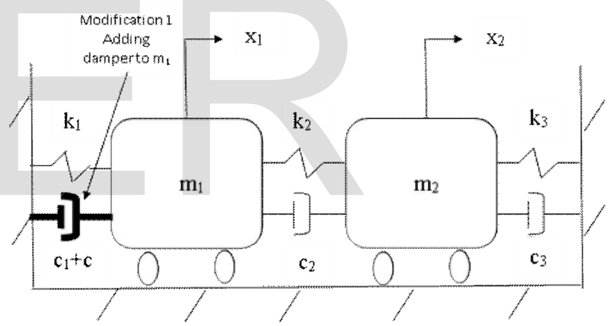


Figure 4: Damping Modification (a) Adding damper to m1 (b) Adding damper to m2 (c) Adding damper between m1, m2

6 RESULTS

The effect of structural Dynamic modifications discussed in above section are analyzed and reported. mass modifications tend to shift the natural frequency of the system and as per requirement and decision of the designer one may choose the appropriate modification. However, the chosen parameter change may not be optimum as this is not considered in the present study.

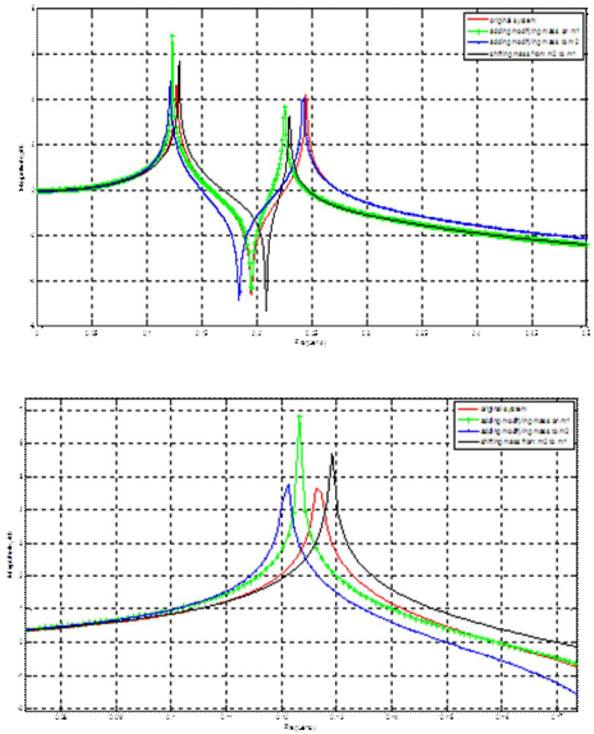
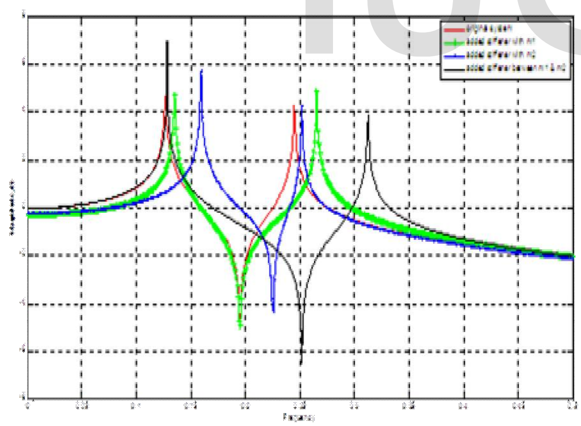


Figure 5: (a) FRF of two dof system with mass modifications (b) close view of first mode clearly depicting modification effects

In case of stiffness modifications, the results are shown on figure 6. It is clear that the adding stiffener between base and m2 is best modification if one wants to shift the resonant frequency on higher side. The third modification has very little effect on resonant frequency and should be avoided.



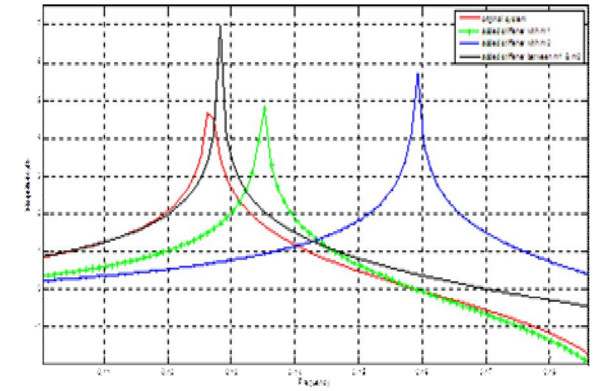


Figure 6: (a) FRF of Two dof system with stiffener modifications (b) close view of first mode clearly depicting modification effects

In case of damping modifications, the results are shown on figure 7. As it is obvious that the natural frequency is governed by stiffness and mass and its distribution in the system so there is no shifting of natural frequency in any of the damping modifications however, the damping affects the peak values only

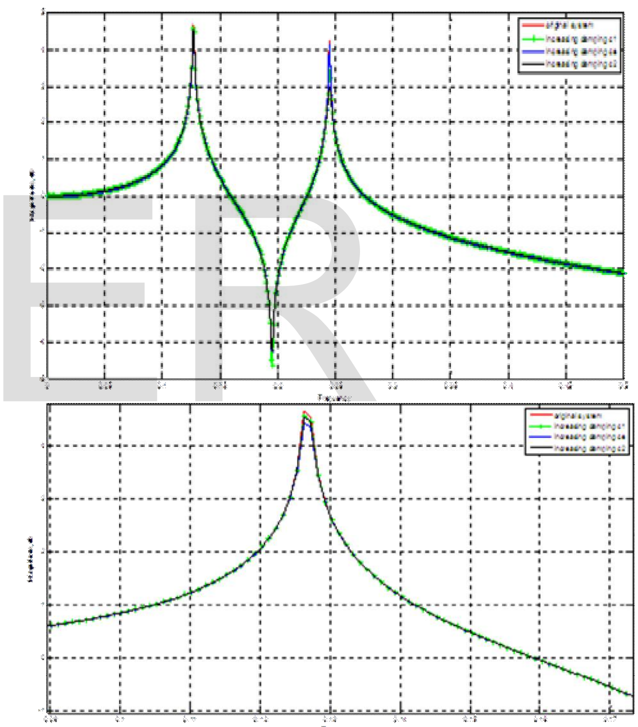


Figure 7: (a) FRF of Two dof system with damping modifications (b) close view of first mode depicting modification effects

4 Conclusion

The numerical example of above case study clearly demonstrates the use of SDM to achieve dynamic design. The SDM is applied on discrete models which are simple but represent case of complex mechanical structures. Model adjustment of discrete (analytical model) system with system identification gives the feasibility of this technique to help achieve SDM at design level thus avoiding prototyping-building-test-modification route. Which in turn can lead to better & economic products.

ACKNOWLEDGMENT

The authors wish to thank Prof. S.M. Muzakkir, Deptt. Of Mechanical Engineering, Jamia Millia Islamia, New Delhi for His support and encouragement.

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